Final Exam Cosmology - Study year 2020-2021

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The exam duration (with additional time for downloading and uploading) is 3.5 hours.
The exam is an open book exam
The number of points given to each question is indicated next to it. The grade will be based on your answers to all questions.
Indicate clearly the steps in your solution and provide sufficient text.

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1. This question is composed of 4 short questions with 12.5 points each.

   (a) **If instead of 3 species of neutrinos there were 4 species, what would be the ratio between the neutrino and photon temperatures today? How do you interpret your answer?**

   (b) **If the difference between the neutron mass and proton mass was** 1.0 **MeV and not** 1.29 **MeV, what implications would that have on Big Bang Nucleosynthesis?**

   (c) **We have shown in the class that the angular diameter distance for the benchmark model has a maximum at redshift of about** 1.5**. This implies that at larger distances a fixed size object will look larger. What in your opinion is the cause for this behavior?**

   (d) **Assume that the our Universe did not go through Decoupling and remained ionized through-out all of its history. Calculate the redshift at which the Universe becomes transparent (assume that baryons in the Universe are only in the form of protons).**

(50 points)

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2. Assume a Universe with $k = 0$ (flat).

- **Write the relation between the time coordinate, t, and the distance coordinate, r, for an observed galaxy.**
  Hint: consider ds^2 for the photons emitted from the galaxy.
  (9 points)
- **In the case in which $(a/a_0) = (t/t_0)^n$, where $n < 1$, show that the distance coordinate of an object seen at redshift z is**

$$r = \frac{ct_0}{(1-n)a_0}\left[1 - (1+z)^{1-1/n}\right]$$

  (8 points)
- **Calculate the present proper distance to a quasar at redshift 5 for the case of $n = 2/3$. Assume you know H_0.**
  (8 points)

(Total of 25 points for the question.)

(25 points)

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3. In the class we have written the Friedmann equation and solved it for several cases. Here we will consider the generic case of matter and dark energy (Λ) Universe.

   - **Write the Friedmann equation for such a Universe but with dark energy equation of state given by generic w and not necessarily $w = -1$.**
     (5 points)
   - **If one ignores the matter components, show that for $w < -1$ the scale factor blows up to infinity at a certain time. Assume that the current time is t_0 and the current scale factor of the Universe is $a_0 = 1$**
     (10 points)
   - **Such behaviour of the scale factor is called the "Big Rip". Calculate the time of the big rip with respect to current time for the case of $w = -3/2$ and $H_0 = 70\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$.**
     (10 points)

(25 points)

Good Luck!